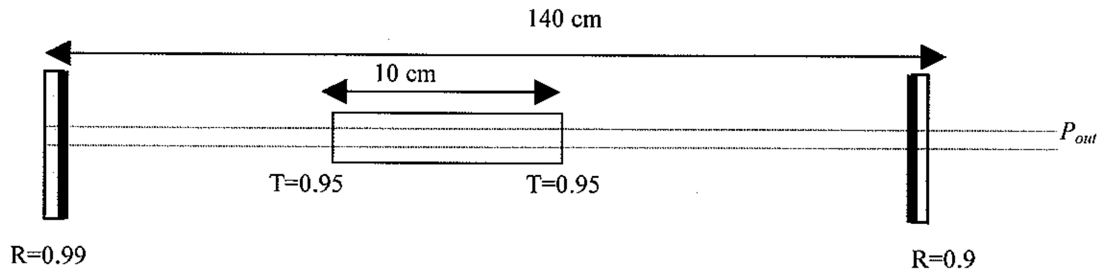
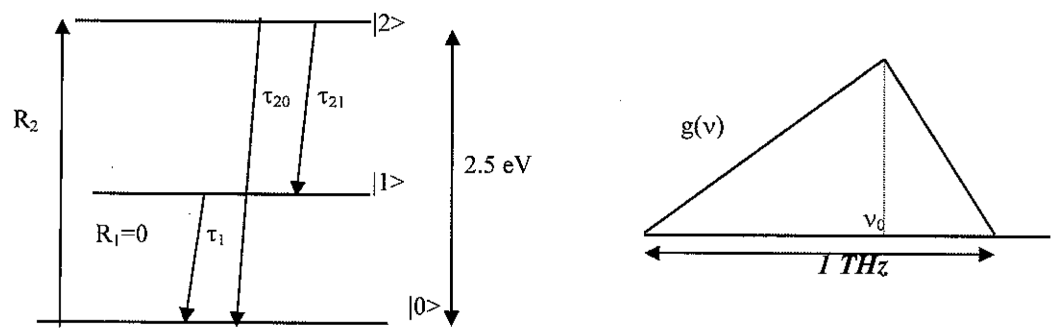
Solution

Laser Physics II (PHYC/ECE 564)
Spring 2008
Homework #1, Due Wednesday Feb. 6

1. Consider the laser system shown in the figure below. The following parameters are known: *Einstein A coefficient:* $5x10^6$ sec^{-1}, *Lifetimes:* $\tau_2=(1/\tau_{12}+1/\tau_{20})^{-1}$ $=10$ μsec, $\tau_1\approx 0$ *(very fast), Wavelength (line center):* $\lambda_0=1$ μm, *Refractive index of the active medium:* $n=1.7$
Degeneracy factors: $g_1=g_2=1$, *Beam Area :* $A=0.1$ mm^2



The energy levels and the homogenous lineshape of the gain medium is approximated by the following diagrams.



(a) What is the quantum efficiency? (0.5 pts.)

(b) Calculate the stimulated emission cross section. (0.5 pts.)

(c) What is the threshold population inversion? (1.5 pts.)

(d) Calculate the threshold excitation rate R_2^{th}. (1.0 pts.)

(e) If R_2 is set at $3\times R_2^{th}$, estimate the CW output power, P_{out}. (2 pts.)

(f) What is the Schawlow-Townes linewidth for this laser assuming the power output obtained in (e) (2.pts.)

(g) If this laser were to be mode-locked, describe the output pulse train. Include an estimation of the shortest pulse width (Δt_p) and the peak output power knowing the average power obtained in (e). (2.5 pts.)

2. Consider a unidirectional c.w. ring laser as depicted in Figure 1-a. Fig. 1-b shows the energy diagram of the *homogeneously broadened* gain medium. The laser power is coupled out and monitored (by a power meter) through an output coupler with transmission $T_2 \ll 1$ (i.e. high-Q cavity). The fluorescence from |2> to |1> transition is also monitored using a detector as shown. An intracavity chopper is used to modulate the output power at a slow rate. Figure 2 depicts the variation of the output power and the florescence signal as the chopper blocks and unblocks the resonator. The effective beam area inside the gain medium is A. The upper-state lifetime is short enough to ignore Q-switching.

(a) Determine the saturation intensity I_s in terms of the measured and given quantities (6 points)

(b) Design an experiment that uses part (a) in order to determine the gain cross section σ_{12}. (4 points)

Figure 1

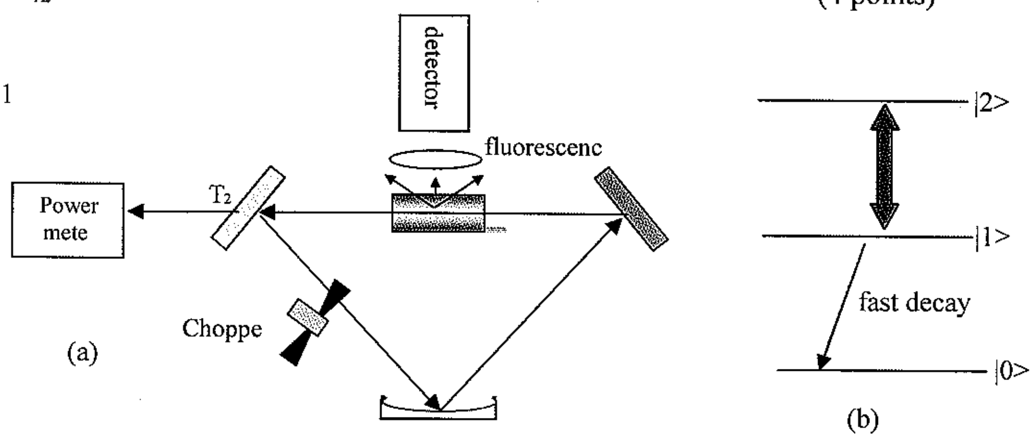
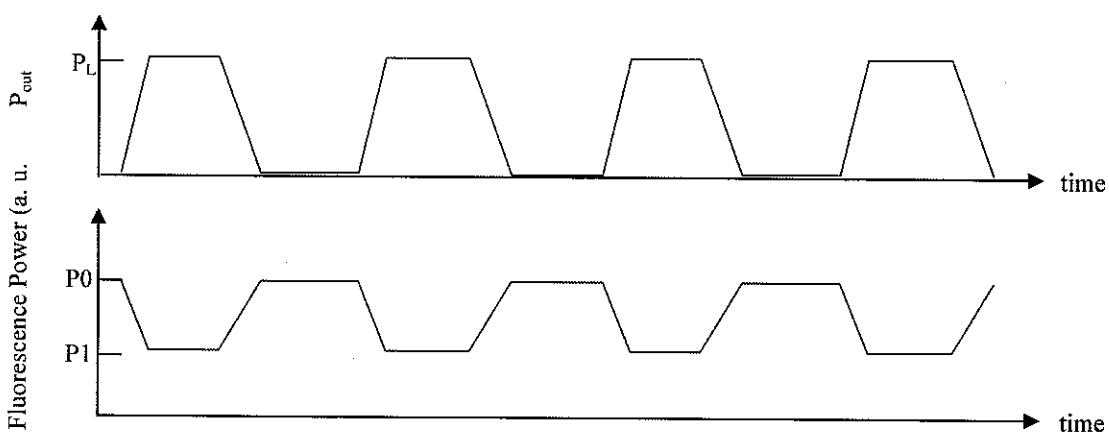


Figure 2



Laser Physics II

Homework #1 — Solution

1 (a)

lasing wavelength $\lambda_0 \doteq 1\,\mu m$

$\therefore \Delta E = \frac{1.24}{1} = 1.24\ eV$

$\therefore \eta_{qe} = \frac{1.24}{2.5} = 49.6\%_{*}$ … quantum efficiency

(b) From the lineshape of the gain medium

$g(\nu_0) = \frac{2}{\Delta\nu}$

$= 2\times 10^{-12}\ s$

$\therefore \sigma(\nu_0) = A_{21}\frac{\lambda_0^2}{8\pi n^2}\, g(\nu_0)$

$= 5\times 10^{6}\ s^{-1}\cdot \frac{\mu m^2}{8\pi\times 1.7^2}\cdot 2\times 10^{-12}\ s$

$= 0.138\times 10^{-6}\ \mu m^2$

$= 1.38\times 10^{-15}\ cm^2{}_{*}$ …. stimulated emission cross section

(c) $R_1 R_2 (T_1 T_2)^2 e^{2\gamma_{th}\cdot l_g} = 1$ … threshold condition

$0.9\cdot 0.99\cdot(0.95\cdot 0.95)^2\, e^{2\cdot 10\cdot \gamma_{th}} = 1$

$\therefore \gamma_{th} = 0.016\ cm^{-1}$

And $\gamma_{th} = \sigma(\nu_0)\,(N_2 - N_1)_{th}$

$0.016 = 1.38\times 10^{-15}\,(N_2 - N_1)_{th}$

$\therefore N_2 - N_1 = 1.16\times 10^{13}\ cm^{-3}{}_{*}$ … threshold population inversion

(d) $\tau_1 \approx 0$

$\therefore N_1 = 0 \qquad \therefore N_2 = 1.16\times10^{13}\ cm^{-3}$

For steady state

$$\frac{dN_2}{dt} = R_2^{th} - \frac{N_2}{\tau_2} = 0$$

$$\therefore R_2^{th} = \frac{N_2}{\tau_2}$$

$$= \frac{1.16\times10^{13}\ cm^{-3}}{10\ \mu sec}$$

$= 1.16\times10^{18}\ cm^{-3}/s$ * ... threshold excitation rate

(e) $R_2 = 3R_2^{th}$

$\therefore \gamma_0 \doteq 3\gamma_{th} = 0.048\ cm^{-1}$

We know $I_s = h\nu/\sigma\tau_2 = \dfrac{1.24\ eV}{1.38\times10^{-15}\ cm^2 \cdot 10^{-5}\ s} = 14.38\ W/cm^2$

$$\gamma_{th}/\gamma_0 = \frac{1}{1+2I_\nu/I_s} = \frac{1}{3}$$

$\therefore I_\nu = I_s = 14.38\ W/cm^2$

$$P_{out} = T(1-R)\, I_\nu \cdot A$$

$$= 0.95 \times 0.1 \times 14.38 \times 0.1 \times 10^{-2}$$

$= 0.0014\ W$ * ..... estimation of CW output power

(f) $$\tau_p = \frac{\tau_{RT}}{1-S}$$

$$= \frac{2nd/c}{1-R_1R_2(T_1T_2)^2}$$

$$= \cancel{\frac{2\times140/3\times10^{10}}{1-0.9\cdot0.99\cdot(0.95)^4}} \qquad \frac{2\times(130+10\times1.7)/3\times10^{10}}{1-0.9\cdot0.99\cdot(0.95)^4}$$

$= 3.4\times10^{-8}\ s$

$= 3.57\times10^{-8}\ s$

$$\therefore \Delta\nu_c \doteq \frac{1}{2\pi\tau_p}$$

$$= \frac{1}{2\cdot 3.14 \cdot \cancel{3.4\times10^{-8}s}\ 3.57\times10^{-8}s}$$

$$= \cancel{4.68\times10^{6}\ s^{-1}}\quad 4.45\times10^{6}\ s^{-1}$$

$$\therefore \Delta\nu_{laser} = \frac{\pi h\nu\,(\Delta\nu_c)^2}{P_{out}}$$

$$= \frac{3.14\times1.24\times1.6\times10^{-19}\,J\ (\cancel{4.68\times10^{6}}\ 4.45\times10^{6})^2\ s^{-2}}{0.0014\ W}$$

$$= \cancel{9.7\times10^{-3}}\ 8.8\times10^{-3}\ Hz \quad \text{Schawlow – Townes linewidth}$$

(g) From the gain spectrum, we ~~may~~ see

$$\Delta\nu = 0.5\ THz$$

$$\therefore \Delta t_p = \frac{0.44}{\Delta\nu}$$

$$= 8.8\times10^{-13}\ s \quad \text{... estimation of pulse width}$$

$$\therefore P_{peak} = P_{out}\cdot\frac{\tau_{RT}}{\Delta t_p}$$

$$= 0.0014\ \cancel{\frac{3.4\times10^{-8}}{3.15\times10^{-13}}}\left(\frac{9.8\times10^{-9}}{8.8\times10^{-13}}\right) = 15.6\ W$$

$$= \cancel{180W} \quad \text{... peak output power}$$

2. (a) Fluorescence Power is proportional to density N_2

When the chopper is closed, N_2 reaches a steady state under the pump.

When the chopper is open, N_2 reaches its threshold value due to saturation, say N_2^{th}

$$\therefore P_0/P_1 = N_2/N_2^{th}$$

$\therefore N_1 = 0$

$\therefore \gamma_0 / \gamma_{th} = N_2 / N_2^{th} = P_0 / P_1$

We know $\gamma_0/\gamma_{th} = 1 + I_\nu / I_s = 1 + \frac{P_L}{T_2 A} / I_s$

$\therefore I_s = \frac{P_L}{T_2 A} \Big/ \left(\frac{P_0}{P_1} - 1 \right)$

(b) $I_s = h\nu / \sigma \tau_2$

In order to determine σ, we should measure ν and τ_2

Send the laser beam to a spectrometer, we can measure ν.

Connect the detector to an oscilloscope, measure the time dependent fluorescence power when the pump is shut down suddenly. From this, we can get τ_2.

Then, we can calculate σ_{12}.